



Over its storied and intriguing history, Hollywood has entertained us with many mathematical moments in film. John Nash in “A Beautiful Mind,” the brilliant janitor in “Good Will Hunting,” the number theory genius in “Pi,” and even Abbott and Costello are just a few of the Hollywood “mathematicians” that come to mind. Here are just a few examples of mathematics in film...

## ▼ *Die Hard: With a Vengeance* - Word Problem



McClane (Bruce Willis) and Zeus (Samuel L. Jackson) find a puzzle with a bomb, beside a fountain in a city park. They are given a five gallon jug and a three gallon jug, and have to measure out exactly four gallons of water on a scale to keep a bomb from exploding. They only have 2 minutes to figure out how to do this...

Can you figure out in two minutes how they did it with just the two buckets? (*Use the buttons below the "buckets" to fill them, pour their contents into the other, or empty them.*)

3 Gallon Bucket	5 Gallon Bucket
<input type="button" value="FILL"/>	<input type="button" value="FILL"/>
<input type="button" value="POUR INTO 5"/>	<input type="button" value="POUR INTO 3"/>
<input type="button" value="EMPTY"/>	<input type="button" value="EMPTY"/>

## ▼ Solution

1. Fill the 3 gallon bucket and then pour it into the 5 gallon bucket
2. Fill the 3 gallon bucket again
3. Pour water from the 3 gallon bucket into the 5 gallon bucket until it is completely full
4. You now have exactly 1 gallon left in the 3 gallon bucket
5. Empty the full 5 gallon bucket
6. Now pour the 1 gallon of water into the empty 5 gallon bucket
7. Fill the 3 gallon bucket again
8. Finally pour those 3 gallons into the 5 gallon bucket which still has 1 gallon in it
9. EXACTLY 4 gallons are now in the 5 gallon bucket

## ▼ Cast Away - Approximation and Area

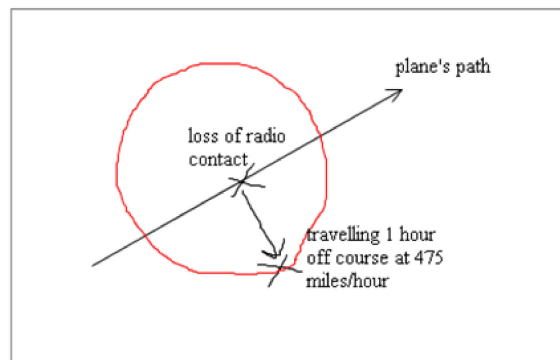


Chuck Noland (Tom Hanks) is flying somewhere over the south Pacific when his plane crashes into the ocean. He floats on a liferaft to a deserted island.

In one scene, he uses the famous formula for the area of a circle,  $A = \pi r^2$  to calculate how large an area his rescuers would have to search.

The plane had been flying from Memphis for 11.5 hours at 475 miles/hour. He estimates that after the plane went out of radio contact, it flew around in a storm for about an hour before crashing, a distance of about 400 miles.

Assuming the search area is centered around the last point of radio contact, he calculates  $Area = \pi r^2$  by approximating  $\pi$  to 3.14:



"...400 miles squared is 160000...times pi..."

$$160000 \cdot 3.14 = 5.0240000 \cdot 10^5$$

This is a very good approximation. We can see Maple do the calculation and approximation:

$$A := \pi r^2$$

evaluate at point  $\rightarrow$

$$\pi r^2$$

(2.1)

$$160000 \pi$$

(2.2)

at 20 digits  
→

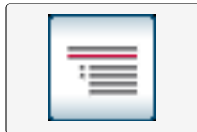
$$5.0265482457436691816 \cdot 10^5$$

(2.3)

Tom estimates that the area is twice that of Texas. Wikipedia gives the area of Texas as 261,797 square miles.

$$261797 \cdot 2 = 523594$$

Again, not a bad estimation!! Maple can calculate  $\pi$ , theoretically, to infinitely many digits. Here are the first 10 000:



#First 10 000 digits of pi...

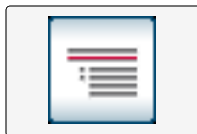
## ▼ *The Fugitive* - Rates and Units



Dr. Richard Kimball (Harrison Ford) has been wrongfully convicted of the murder of his wife, and is being transported to death row in a bus with other inmates. An escape is attempted by the others, and the bus jumps a guardrail and crashes onto a set of train tracks. Kimball escapes from the scene and is on the run.

He is pursued by Deputy US Marshall Samuel Gerard (Tommy Lee Jones). Arriving at the wreckage of the bus,

Gerard states, "Listen up, ladies and gentlemen. Our fugitive has been on the run for ninety minutes. Average foot speed over uneven ground barring injuries is 4 miles an hour and that gives us a radius of six miles." Is he correct?



#Initialization

We can use the rate equation for this question:

$$(\text{Rate of Travel}) \cdot (\text{Time Traveled}) = \text{Distance Covered}$$

Where the rate is  $4 \left[ \frac{\text{mile}}{\text{h}} \right]$  and the time is 90 [min]. Normally, you would need to manually convert one of these values such that the units math, but we can simply assign the values, and their units, to names in Maple:

$$\begin{aligned} \text{Time} &:= 90 \text{ } \llbracket \text{min} \rrbracket : \\ \text{Rate} &:= 4 \left\llbracket \frac{\text{mile}}{\text{h}} \right\rrbracket : \end{aligned}$$

... and substitute into the rate equation to solve for distance:

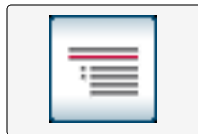
$$\text{Rate} \cdot \text{Time} = 31680 \text{ } \llbracket \text{ft} \rrbracket \xrightarrow{\text{replace units}} 6 \text{ } \llbracket \text{mi} \rrbracket$$

Maple automatically simplifies the units in our calculation and returns an appropriate result. Using the right-click menu we can convert from feet to miles for the final answer.

## ▼ Speed - Projectile Motion and the Range Equation



A city bus in L.A. has been rigged with a bomb that arms when the speed exceeds 50 miles/hour and will explode if it then drops below 50 miles/hour. While driving on the freeway, the bus is forced to jump a 50 foot gap where the road is unfinished. Being Hollywood, of course the jump is successful, but just how realistic is this?



#Initialization

The bus's jump is simply an example of projectile motion: the motion of an object projected into the air at an angle. Other examples include a soccer ball being kicked or a baseball being hit into the air.

The *range equation* is used to calculate the horizontal distance covered by a projectile (neglecting air resistance):

$$R = \frac{V_0^2 \sin(2\theta)}{g}$$

$$R = \frac{V_0^2 \sin(2\theta)}{g} \quad (4.1)$$

where  $V_0$  is the projectile's initial speed,  $\theta$  is the angle the projectile launches at, and  $g$  is of course the acceleration due to gravity,  $32.174 \text{ ft/s}^2$ . From the movie, we know that the length of the gap is 50 feet, and the initial speed is 68 miles/hour. From the equation we can see that if the launch angle is zero, the range will also become zero. It is necessary for the bus to launch at some angle greater than zero to make the jump.

We can isolate the launch angle,  $\theta$ , in the range equation:

$$R = \frac{V_0^2 \sin(2\theta)}{g} \xrightarrow{\text{isolate for theta}} \theta = \frac{1}{2} \arcsin\left(\frac{Rg}{V_0^2}\right)$$

...assign values to the variables:

$$g := 32.174 \left[ \frac{ft}{s^2} \right] :$$

$$R := 50 \left[ ft \right] :$$

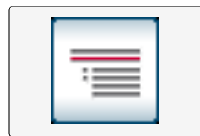
$$V_0 := 68 \left[ \frac{mi}{h} \right] :$$

...and solve numerically for  $\theta$ :

$$\theta := \frac{1}{2} \arcsin\left(\frac{Rg}{V_0^2}\right)$$

0.08122245880 **(4.2)**

By default Maple works in radians but we can easily convert to degrees:



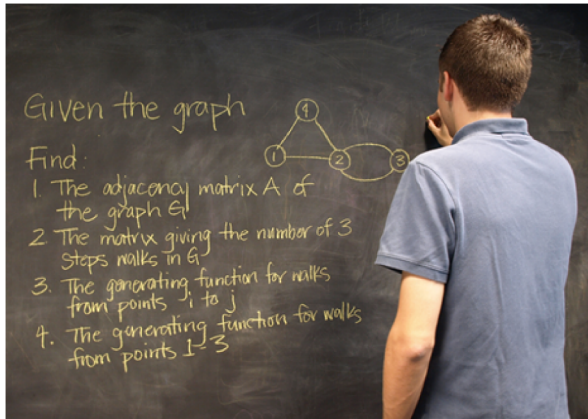
#Convert to degrees

4.653704088 *degrees*

In theory, then, if the bus left the road at an angle of approximately 5 degrees, it would be possible for the bus to jump the gap!

It is interesting to note that the Wikipedia page about this movie states that a ramp was used to enable the bus to jump the 50 feet. The bus was to jump over a regular section of highway, which was edited out of the film with some careful CGI. The director mentions in the DVD commentary that as much as possible was removed from the bus to make it as light as possible. On the first attempt, the bus driver missed the landing, crashing the bus. On the second attempt, the bus was only expected to travel 20 feet, and cameras were placed accordingly. However the bus traveled much further than expected and landed on top of the camera: fortunately a second camera was also filming.

## ▼ *Good Will Hunting* - Linear Algebra and Graph Theory

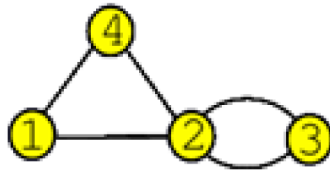


Although Will Hunting (Matt Damon) works as a janitor at MIT, he has genius-level intelligence and a great aptitude for mathematics. Professor Gerard Lambeau (Stellan Skarsgard) leaves a challenge problem on a corridor blackboard for his students, hoping one of them will solve it by semester's end. He describes this problem as an "advanced Fourier System", but it is in fact a problem in algebraic graph theory, to be solved in four stages (shown below). Will solves the problem the next night.

What can Maple do with this problem?

### Problem Statement:

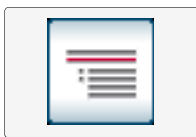
Given the graph



Find:

1. The adjacency matrix  $A$  of the graph  $G$
2. The matrix giving the number of 3 step walks in  $G$
3. The generating function for walks from points  $i$  to  $j$
4. The generating function for walks from points 1-3

### ▼ Solution



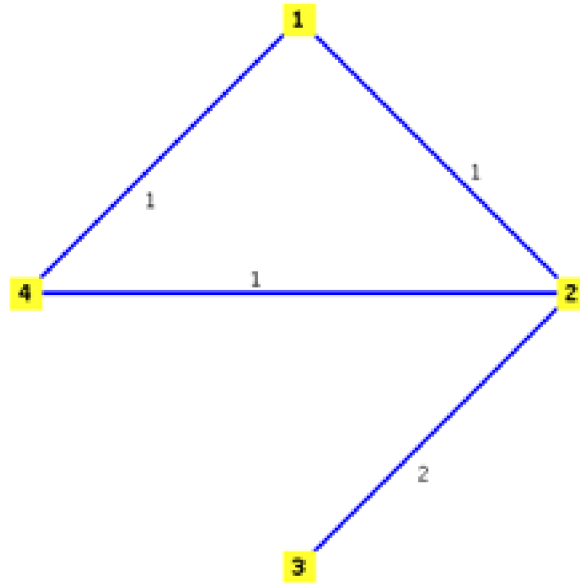
#Initialization

We will use Maple's [GraphTheory](#) package to solve this problem.

### ▼ *Part 1 - The adjacency matrix $A$ of the graph $G$*

Create a weighted graph using Maple's GraphTheory package:

$G := \text{Graph}(\{[\{1, 2\}, 1], [\{1, 4\}, 1], [\{2, 3\}, 2], [\{2, 4\}, 1]\}); \text{DrawGraph}(G)$   
*Graph 1: an undirected weighted graph with 4 vertices and 4 edge(s)* (5.1.1.1)



The *WeightMatrix* command will return the adjacency matrix, A:  
 $A := \text{WeightMatrix}(G)$

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad (5.1.1.2)$$

## ▼ Part 2 - The matrix giving the number of 3 step walks in G

To start, let's look at the number of 2 step walks in G. Here,  $M$  is a symbolic representation of the adjacency matrix:

$$M := \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} \end{bmatrix} :$$

... where  $M$  essentially counts the number of walks of length 1 between vertices.

$[M^2]_{ij}$  is by definition the matrix product of the sum  $M_{i1}M_{1j} + M_{i2}M_{2j} + \dots + M_{in}M_{nj}$ :

$$N := M^2$$

$$\left[ \left[ m_{1,1}^2 + m_{1,2}m_{2,1} + m_{1,3}m_{3,1} + m_{1,4}m_{4,1}, m_{1,1}m_{1,2} + m_{1,2}m_{2,2} + m_{1,3}m_{3,2} \right. \right. \quad (5.1.2.1)$$

$$\begin{aligned}
& + m_{1,4} m_{4,2} m_{1,1} m_{1,3} + m_{1,2} m_{2,3} + m_{1,3} m_{3,3} + m_{1,4} m_{4,3} m_{1,1} m_{1,4} \\
& + m_{1,2} m_{2,4} + m_{1,3} m_{3,4} + m_{1,4} m_{4,4} \Big], \\
& \Big[ m_{2,1} m_{1,1} + m_{2,2} m_{2,1} + m_{2,3} m_{3,1} + m_{2,4} m_{4,1} m_{1,2} m_{2,1} + m_{2,2}^2 + m_{2,3} m_{3,2} \\
& + m_{2,4} m_{4,2} m_{2,1} m_{1,3} + m_{2,2} m_{2,3} + m_{2,3} m_{3,3} + m_{2,4} m_{4,3} m_{2,1} m_{1,4} \\
& + m_{2,2} m_{2,4} + m_{2,3} m_{3,4} + m_{2,4} m_{4,4} \Big], \\
& \Big[ m_{3,1} m_{1,1} + m_{3,2} m_{2,1} + m_{3,3} m_{3,1} + m_{3,4} m_{4,1} m_{3,1} m_{1,2} + m_{3,2} m_{2,2} \\
& + m_{3,3} m_{3,2} + m_{3,4} m_{4,2} m_{1,3} m_{3,1} + m_{2,3} m_{3,2} + m_{3,3}^2 + m_{3,4} m_{4,3} m_{3,1} m_{1,4} \\
& + m_{3,2} m_{2,4} + m_{3,3} m_{3,4} + m_{3,4} m_{4,4} \Big], \\
& \Big[ m_{4,1} m_{1,1} + m_{4,2} m_{2,1} + m_{4,3} m_{3,1} + m_{4,4} m_{4,1} m_{4,1} m_{1,2} + m_{4,2} m_{2,2} \\
& + m_{4,3} m_{3,2} + m_{4,4} m_{4,2} m_{4,1} m_{1,3} + m_{4,2} m_{2,3} + m_{4,3} m_{3,3} + m_{4,4} m_{4,3} \\
& m_{1,4} m_{4,1} + m_{2,4} m_{4,2} + m_{3,4} m_{4,3} + m_{4,4}^2 \Big] \Big]
\end{aligned}$$

The number of paths of length 2 between nodes 1 and 3 is  $[M^2]_{13}$ , or  $[N]_{13}$ :

$(N) [1, 3]$

$$m_{1,1} m_{1,3} + m_{1,2} m_{2,3} + m_{1,3} m_{3,3} + m_{1,4} m_{4,3} \quad (5.1.2.2)$$

This formula counts the number of walks of length 2 from vertex 1 to 3:

(# walks of length 1 from  $1 \rightarrow 1$ )  $\times$  (# walks of length 1 from  $1 \rightarrow 3$ ) +  
 (# walks of length 1 from  $1 \rightarrow 2$ )  $\times$  (# walks of length 1 from  $2 \rightarrow 3$ ) +  
 (# walks of length 1 from  $1 \rightarrow 3$ )  $\times$  (# walks of length 1 from  $3 \rightarrow 3$ ) +  
 (# walks of length 1 from  $1 \rightarrow 4$ )  $\times$  (# walks of length 1 from  $4 \rightarrow 3$ )

We can see that similarly,  $[M^n]_{ij}$  is the number of paths of length n going from i to j.

For instance, it is easy to see that walks of length 3 are represented by  $M^3$ . The formula for the number of walks of length 3 from points 1 to 3 is:

(# walks of length 1 from  $1 \rightarrow 1$ )  $\times$  (# walks of length 2 from  $1 \rightarrow 3$ ) +  
 (# walks of length 1 from  $1 \rightarrow 2$ )  $\times$  (# walks of length 2 from  $2 \rightarrow 3$ ) +  
 (# walks of length 1 from  $1 \rightarrow 3$ )  $\times$  (# walks of length 2 from  $3 \rightarrow 3$ ) +  
 (# walks of length 1 from  $1 \rightarrow 4$ )  $\times$  (# walks of length 2 from  $4 \rightarrow 3$ )

The matrix representing the number of walks from points  $i$  to  $j$  of length 3 for our graph G is  $A^3$ :

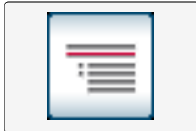


$A^3$

$$\begin{bmatrix} 2 & 7 & 2 & 3 \\ 7 & 2 & 12 & 7 \\ 2 & 12 & 0 & 2 \\ 3 & 7 & 2 & 2 \end{bmatrix}$$

(5.1.2.3)

### ▼ Part 3 - The generating function for walks from points $i$ to $j$



#Initialization

Here is a generating function for the number of walks of length  $k$ , expressed as an expression for the summation of a geometric series:

$$F := \sum_{k=0}^{\infty} M^k \cdot z^k = - \frac{1}{Mz - 1}$$

where  $M$  is the adjacency matrix and  $z$  is a placeholder.

This formula also holds for matrices!

$$[F(z)]_{ij} = \sum_{n=0}^{\infty} [M^n]_{ij} z^n = \left[ \sum_{n=0}^{\infty} M^n z^n \right]_{ij} = [(1 - Mz)^{-1}]_{ij}$$

For our graph  $G$ , we can write:

$$K := (1 - Az)^{-1}$$

$$\begin{aligned} & \left[ \left[ -\frac{-1 + 5z^2}{4z^4 - 2z^3 - 7z^2 + 1}, \frac{z}{4z^3 - 6z^2 - z + 1}, \frac{2z^2}{4z^3 - 6z^2 - z + 1}, \right. \right. \\ & \quad \left. \left. - \frac{z(4z^2 - z - 1)}{4z^4 - 2z^3 - 7z^2 + 1} \right], \right. \\ & \quad \left[ \frac{z}{4z^3 - 6z^2 - z + 1}, -\frac{-1 + z}{4z^3 - 6z^2 - z + 1}, -\frac{2z(-1 + z)}{4z^3 - 6z^2 - z + 1}, \right. \\ & \quad \left. \frac{z}{4z^3 - 6z^2 - z + 1} \right], \\ & \quad \left[ \frac{2z^2}{4z^3 - 6z^2 - z + 1}, -\frac{2z(-1 + z)}{4z^3 - 6z^2 - z + 1}, -\frac{(2z - 1)(1 + z)}{4z^3 - 6z^2 - z + 1}, \right. \\ & \quad \left. \frac{2z^2}{4z^3 - 6z^2 - z + 1} \right] \end{aligned}$$

(5.1.3.1)

$$\left[ -\frac{z(4z^2 - z - 1)}{4z^4 - 2z^3 - 7z^2 + 1}, \frac{z}{4z^3 - 6z^2 - z + 1}, \frac{2z^2}{4z^3 - 6z^2 - z + 1}, -\frac{-1 + 5z^2}{4z^4 - 2z^3 - 7z^2 + 1} \right]$$

#### ▼ Part 4 - The generating function for walks of length $k$ from points 1-3

The generating function for the number of paths of length  $k$  from point 1 to 3 is thus:

$K[1, 3]$

$$\frac{2z^2}{4z^3 - 6z^2 - z + 1} \quad (5.1.4.1)$$

series in  $z$  →

$$2z^2 + 2z^3 + 14z^4 + 18z^5 + 94z^6 + 146z^7 + 638z^8 + 1138z^9 \quad (5.1.4.2)$$

We are using Maple's *series* command to expand the generating function (5.1.4.1) back into an infinite series. The coefficient of  $z^k$  gives the number of walks of length  $k$ . For instance, the 1st 2 terms are zero as there are no paths of length 0 or 1.

We can simply look at the coefficients of the series expansion terms to determine the number of walks of length  $k$  between points 1 and 3.

Calculate # Walks

Clear All

There are  walks of length  between points  and .

## ▼ References

Allen, D. (2004). *Math in the Movies*. Retrieved August 20, 2008, from <http://www.math.tamu.edu/~dallen/hollywood/mathindex.htm>.

Armstrong, S., et al (Executive Producers), and Van Sant, G. (Director). (1997). *Good Will Hunting* [Motion picture]. New York, NY: Miramax Films.

Barish, K., Huggins, R. (Executive Producers), and Davis, A. (Director). (1993). *The Fugitive* [Motion picture]. Burbank, CA: Warner Brothers.

Bradshaw, J. (Executive Producer), and Zemeckis, R. (Director). (2000). *Cast Away* [Motion picture]. Century City, CA: Twentieth Century-Fox Film Corporation.

Bryce, I. (Executive Producer), and de Bont, J. (Director). (1994). *Speed* [Motion picture]. Century City, CA: Twentieth Century-Fox Film Corporation.

Colarusso, D. (2007, March). *Could that actually happen? Phylm examines the bus jump from Speed*. Retrieved August 20, 2008, from <http://www.davidcolarusso.com/blog/?p=32>.

Feitshans, B., Lawrence, R. (Executive Producers), and McTiernan, J. (Director). (1995). *Die Hard: With a Vengeance* [Motion picture]. Century City, CA: Twentieth Century-Fox Film Corporation.

Hornsby, J. (2007). *Math Class goes to Hollywood*. Retrieved August 20, 2008, from <http://www.geocities.com/johnnyhornsby/mathclassgoestohollywood>.

Jagers, B. (2001, June). *Mathematics in the movie "Good Will Hunting"*. Retrieved August 20, 2008, from The Maple Application Center: [http://www.maplesoft.com/applications/app\\_center\\_view.aspx?AID=77](http://www.maplesoft.com/applications/app_center_view.aspx?AID=77).

Knill, O. (2008, July). *Mathematics in Movies*. Retrieved August 20, 2008, from <http://www.math.harvard.edu/~knill/mathmovies>.

Reinhold, A. (2007). *The Math in the Movies Page*. Retrieved August 20, 2008, from <http://world.std.com/~reinhold/mathmovies.html>.

*Legal Notice: The copyright for this application is owned by Maplesoft. The application is intended to demonstrate the use of Maple to solve a particular set of problems. It has been made available for product evaluation purposes only and may not be used in any other context without the express permission of Maplesoft.*